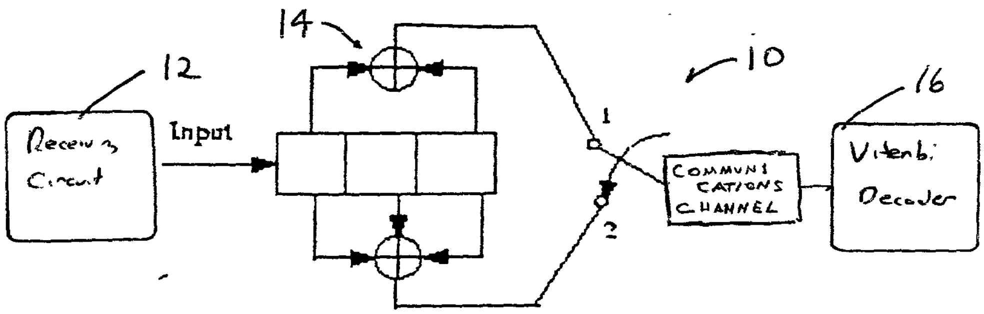

Figure 1 $K = 3, l = 2$ convolutional encoder

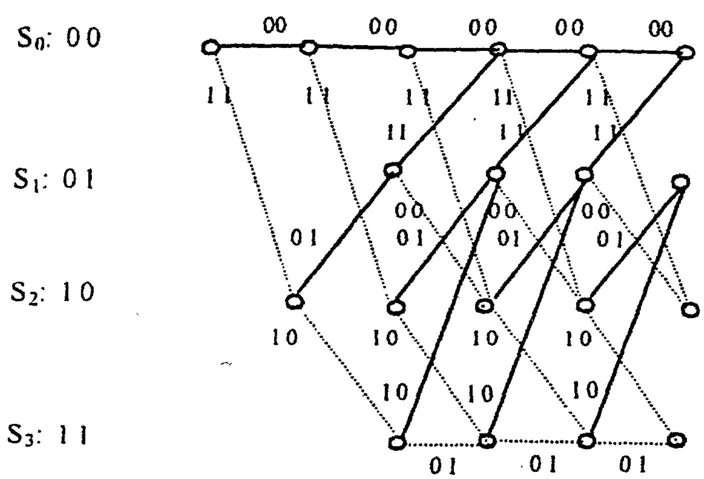

Figure 2 – Trellis for ordinary convolutional code

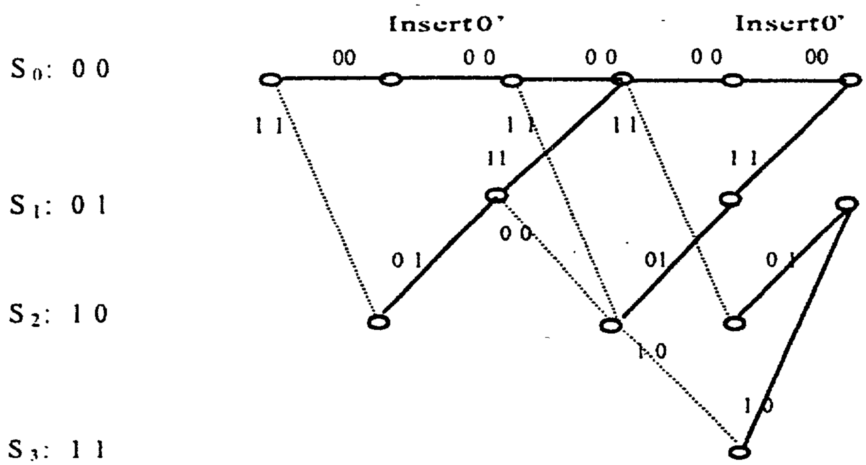

Figure 3 – Inserting zero at the first position periodically

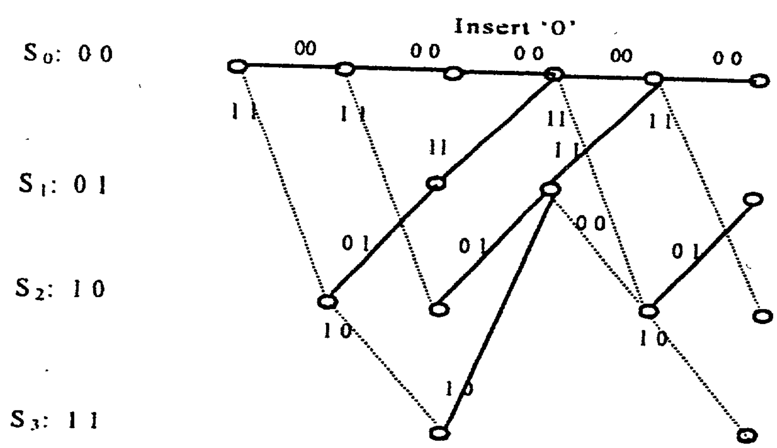

Figure 4 – Insert zero at the second position periodically

Generator Matrix

Let

$$C^{(j)} = X^{(j)} G, \quad j = 1, 2, ..., K-1, \qquad (1)$$

where $X^{(j)} = [1, x_1, ..., x_{j-1}, 0, x_{j+1}, ...], x_{tK+j} = 0, t = 0, 1, ...,$ G is the Toeplitz block matrix

$$G = [\overrightarrow{g}_{i-j}]_{i,j=0,1,....}$$

with $1 \times K$ sub-blocks

$$\overrightarrow{g}_i = \begin{cases} [g_{1,i}, g_{2,i}, ..., g_{l,i}], & i = 0, 1, ..., m; \\ 0, & \text{others.} \end{cases}$$

FIG. 5

FIG. 6 G_j Presentation

$$\begin{bmatrix} \overrightarrow{g}_0(t) & \overrightarrow{g}_1(t+1) & \cdots & \overrightarrow{g}_m(t+m) & \cdots \\ 0 & \overrightarrow{g}_0(t+1) & \cdots & \overrightarrow{g}_{m-1}(t+m) & \cdots \\ \cdot & \cdot & \cdots & \cdot & \cdots \end{bmatrix},$$

$$\begin{aligned} & \phi_t\left(X_{t-K+2}^t\right) \\ = & \max_{X_0^{t-K+1}} M\left(X_0^t\right) \\ = & \max_{x_{t-K+1}} \left[L\left(X_{t-K+1}^t\right) + \phi_{t-1}\left(X_{t-K+1}^{t-1}\right)\right] \end{aligned}$$

FIG. 6A

DECODING

Step 1 Initialization: For $0 \leq t < K-1$, starting from $\phi\left(X_{-K}^{-1}\right) = 0$ we calculate $\phi\left(X_{t-K+1}^{t}\right)$ for all possible combinations of X_0^t by (3).

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Step 2 Recursive forward algorithm at t:
If $t \neq K-1 \,(\mathrm{mod}\ K)$, we compute $\phi\left(X_{t-K+2}^{t}\right)$ by (3) and save

$$
\begin{aligned}
&\widetilde{x}_{t-K+1}\left(X_{t-K+2}^{t}\right) \\
&= \arg\max_{x_{t-K+1}} \left[L\left(X_{t-K+1}^{t}\right) + \phi\left(X_{t-K+1}^{t-1}\right)\right] \quad (5)
\end{aligned}
$$

otherwise we compute $\phi\left(X_{t-K+2}^{t}\right)$ by (4).
Go to Step 3.

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STEP 3

FIG 7A

Step 3 Recursive backward algorithm at t:
If $t - D \neq K - 1 \pmod{K}$, starting from

$$\hat{X}_{t-K+2}^{t} = \arg \max_{X_{t-K+2}^{t}} \phi\left(X_{t-K+2}^{t}\right) \qquad (6)$$

we calculate $\hat{x}_k = \widetilde{x}_k\left(\hat{X}_{k+1}^{k+K-1}\right)$, $k = t-K+1, t-K, t-K-1, \ldots$ until backward D symbols to find

$$\hat{x}_{t-D} = \widetilde{x}_{t-D}\left(\hat{X}_{t-D+1}^{t-D+K-1}\right); \qquad (7)$$

otherwise $\hat{x}_{t-D} = 0$.

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$T \neq N$, Back to Step 2

If $t = n$ go to Step 4; otherwise go to Step 2.

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Step 4 Termination: Let $n \leq t < n + K - 2 = N$.

If $t \neq K - 1 \pmod{K}$, we compute $\phi\left(X_{t-K+2}^{t}\right)$ by (3) and save $\widetilde{x}_{t-K+1}\left(X_{t-K+2}^{t}\right)$ by (5); otherwise we compute $\phi\left(X_{t-K+2}^{t}\right)$ by (4) and we do not need to save $\widetilde{x}_{t-K+1}\left(X_{t-K+2}^{t}\right)$ since it must be zero.

Repeat this step until $t = N$, then go to Step 5.

To Step 5

FIG 7B

From Step 4

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Step 5 Recursive backward algorithm at the end: Starting from

$$\hat{x}_n = \arg\max_{x_n} \phi\left(\underbrace{0, ..., 0}_{K-2}, x_n\right),$$

we estimate x_t by

$$\hat{x}_t = \tilde{x}_t\left(\hat{X}_t^{t+K-2}\right), \quad t = n-1, n-2, ..., n-D.$$

FIG. 7C

Code	Conv. Code	Conv. Zero Code
Code Rate	$\frac{T}{(T+K-1)l} \approx \frac{1}{l}$	$\frac{T}{Nl} \approx \frac{K-1}{Kl}$
Complexity	$\approx T(l+2)\,2^K$	$\approx \frac{K}{K-1} T(l+2)\,2^{K-1}$
Memory	$2^K D$	$2^{K-1}\left(D - \left[\frac{D}{K}\right]\right)$
Delay	D	D

FIG 8